

## Things to know for Exam 3

### 1. GENERAL KNOWLEDGE

- (1) All material from the first two tests.

### 2. FUNCTIONS

- (1) Be able to determine the domain and image of a relation.
- (2) Be able to determine when a relation is well-defined.
- (3) Know what a function is.
- (4) Be able to prove that two functions are equal (or not equal).
- (5) Be able to prove statements about injectivity, surjectivity, bijectivity.
- (6) Be able to find the image/range of simple functions.
- (7) Determine if the inverse relation to a function is a function.
- (8) Be able to compute inverse functions.
- (9) Know how to work with characteristic functions.
- (10) Know the definitions of images and preimages of sets.
- (11) Be able to work with piecewise defined functions.
- (12) Use the Pasting-together theorem to create bijections.
- (13) Be able to prove Theorems 27.11, 27.15, and 27.20.

### 3. CARDINALITY

- (1) Know the meaning of  $|A| = |B|$ ,  $|A| \leq |B|$ , and  $|A| < |B|$ .
- (2) Be able to prove that  $|\mathbb{N}| = |\mathbb{Z}|$ .
- (3) Know examples of countable, countably infinite, and uncountable sets.
- (4) Be able to use Theorems 29.14 and 30.3 to prove that sets are countable.
- (5) Know how to prove Theorem 30.1, 30.3, 30.5, 31.2, 32.5.
- (6) Know how to use the Schröder-Bernstein Theorem.

### 4. DEFINITIONS TO KNOW

- (1) Domain of a relation (24.1)
- (2) Image of a relation (24.1)
- (3) Well-defined (24.11)
- (4) Function (24.15)
- (5) Identity Function (24.18)
- (6)  $f : A \rightarrow B$  (25.1)
- (7) Codomain (25.1)
- (8) Inverse relation (25.7)
- (9) Image of  $S$  (25.11)
- (10) Preimage of  $T$  (25.14)
- (11) Injective (26.1)
- (12) Surjective (26.13)
- (13) Bijective (26.27)
- (14) Inverse function (Section 27.C)
- (15) Composition of functions (27.1)
- (16) Characteristic function (28.13)
- (17)  $f|_S$  (28.16)
- (18) Having the same cardinality. (29.1)
- (19) Having different cardinality. (29.1)
- (20) Countably infinite (29.9)
- (21) Countable (29.10)
- (22) Uncountable (31.1)
- (23)  $|A| \leq |B|$  (32.1)
- (24)  $|A| < |B|$  (32.1)
- (25)  $\mathcal{F}(S)$  (page 238)