Math 372 Final Review

Below is a summary of what you should know from each chapter of the textbook. You should be familiar with and be able to use all listed theorems; you should **know how to prove theorems in boldface**. You should review the listed homework problems, and know how to solve them. At the end of the review sheet is a list of definitions that you may be asked to give on the final.

Chapter 1: You will not be required to use Cardan's formula to solve cubics. You should be familiar with the definition of the discriminant (at the beginning of section 1.3) and know Theorem 1.3.1.

Chapter 2: You should be able to do basic arithmetic with multivariable polynomials, and understand what symmetric polynomials are. Know how to write a symmetric polynomial in terms of elementary symmetric polynomials.

Theorems: Cor. 2.1.5, Thm 2.2.2, Cor 2.2.5.
Homework: 2.2: 10, 11, 18

Chapter 3: Know the basics of ring theory from Math 371; ideals, prime ideals, maximal ideals, irreducible polynomials. Given a field F and an polynomial $f \in F[x]$, know how to construct a field extension L/F containing a root of f. Know the Fundamental Theorem of Algebra, and what an algebraically closed field is.

Theorems: **Prop. 3.1.3**, Thm 3.1.4, Theorem 3.2.4.
Homework: 3.1: 5; 3.2: 7.

Chapter 4: Know what an extension field is, and how to construct one. Understand the meaning of $F(\alpha_1, \ldots, \alpha_n)$. Know what irreducible polynomials are, and be able to use Eisenstein's criterion to prove a polynomial is irreducible. Know the meaning of finite, algebraic, transcendental, as they relate to field extensions and elements of fields. Know what the degree of an extension is, and know the tower theorem.

Theorems: **Lemma 4.1.3, Lemma 4.1.9**, Thm. 4.2.3 (Eisenstein's Criterion), Lemma 4.3.4, Lemma 4.3.4, **Thm. 4.3.8 (Tower theorem)**, Lemma 4.4.2, Cor. 4.4.9
Homework: 4.2:8; 4.3: 2, 5; 4.4: 2

Chapter 5: Know what splitting fields, normal extensions and separable extensions are, and how they relate. Be able to know when the Theorem of the Primitive Element applies.

Theorems: Prop. 5.1.8, Thm. 5.2.4, Prop. 5.3.2, **Theorem 5.3.5 (separability)**.
Homework: 5.1: 5, 6, 10, 13; 5.2: 1, 3(ab); 5.3: 3.

Chapter 6: For an extension L/F, know what the Galois group is, how to construct elements of the Galois group, how to write the Galois group of a splitting field of f as a permutation group on the roots of f. Be familiar with the computation of the Galois group in Section 6.4(C).

Theorems: Lemma 6.1.3, Prop. 6.1.4, **Theorem 6.2.1**, Prop. 6.3.7,
Homework: 6.1: 2; 6.2: 2, 6; 6.3: 1, 4 6.4: 13

Chapter 7: Know what a **Galois Extension** is. Know the relationship between normal subgroups of the Galois group and normal subfields. Know the Fundamental Theorem of Galois Theory (both parts).

Theorems: **Theorem 7.1.1 (Definition of a Galois Extension)**, Prop. 7.1.3, Lemma 7.2.4, Theorem 7.2.7, Thm. 7.3.1 (Fund. Thm. Part 1), Theorem 7.3.2(Fund. Thm. Part 2), Prop 7.4.2
Homework: 7.1: 1, 3, 11; 7.2: 11; 7.3: 6; 7.4: 2

Chapter 8: Know what solvable groups and radical and solvable extensions are. Know how these concepts relate to each other, and to the problem of solving polynomials by radicals.

Theorems: Thm. 8.1.4, Prop. 8.1.5, Prop 8.2.6, Lemma 8.2.7, Lemma 8.3.2, Thm. 8.3.3, Thm. 8.4.5, Thm. 8.5.3
Homework: 8.1: 1, 8; 8.2: 3, 6, 7; 8.3: 2; 8.4: 4; 8.5: 2

Chapter 9: Know what cyclotomic polynomials and cyclotomic extensions are, and know the Galois group of a cyclotomic extension over $\mathbb{Q}$.
Theorems: Lemma 9.1.1, Prop. 9.1.5, Theorem 9.1.9,
Theorem 9.1.11 (Galois group of cyclotomic extension)
Homework: 9.1: 3, 7, 10, 11, 16

Chapter 10: Know the basic steps allowed in ruler and compass construction, and how these can be related to arithmetic operation. Be able to prove that constructible numbers form a field, and that the square root of a constructible number is constructible. Be able to identify numbers that are not constructible.
Theorems: **Thm. 10.1.4 (Constructible numbers are a field)**, Theorem 10.1.6
Homework: None.

Chapter 11: Know the structure of finite fields, and that every finite extension of finite fields is Galois. Be able to compute all subfields of a finite field. Be able to compute the number of irreducible polynomials of a given degree over a finite field.
Theorems: **Prop. 11.1.1 (Finite fields have prime power order)**, Cor. 11.1.3, Thm. 11.1.4, Thm. 11.1.7, **Thm 11.1.2 (Finite fields are splitting fields)**, Thm. 11.2.4
Homework: 11.1: 11; 11.2: 1, 2, 3

Definitions to know:

Elementary Symmetric Polynomials (28)	Symmetric polynomial (30)	Discriminant (46)
Algebraic element (73)	Transcendental element (73)	Minimal polynomial (74)
$F(\alpha_1, \ldots, \alpha_n)$ (76)	Finite extension (89)	Degree of an extension (89)
Algebraic extension (95)	Splitting field (101)	Normal extension (108)
Separable polynomial (109)	Separable element (111)	Separable extension (111)
Primitive element (119)	Gal(L/F) (125)	Gal(f) (128)
Fixed field (147)	L/F **is Galois** (149)	Galois closure (152)
Conjugate fields (154)	Solvable group (191)	Radical extension (197)
Solvable extension (197)	Compositum (198)	Simple group (210)
Expressible by radicals (215)	Solvable by radicals (215)	Euler ϕ function (230)
Cyclotomic polynomial Φ_n (231)	Constructible numbers (255)	Frobenius automorphism (295)