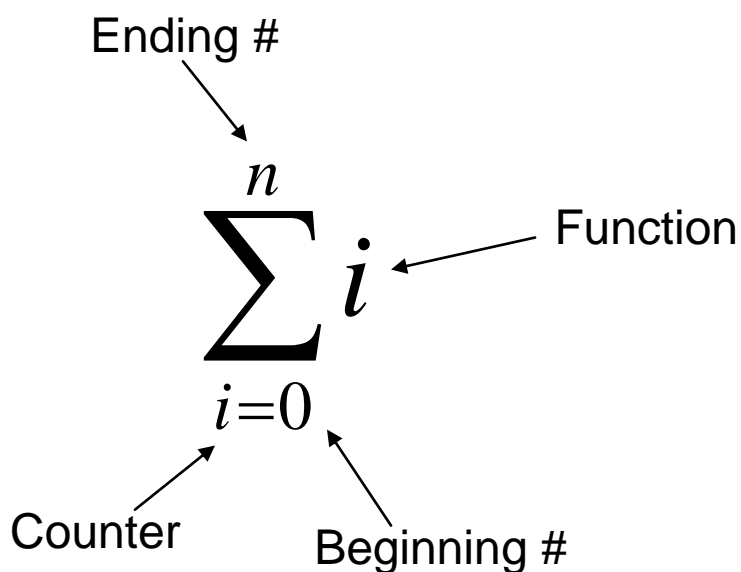


Summation Notation



To evaluate a summation symbol:

1. Note what your beginning number and ending number are.
2. Evaluate the function by plugging in the beginning number to the counter variable.
3. Next you will add that value to the function evaluated at the integer that is one more than the beginning value.
4. Continue adding these function values, each time plugging in one number greater than the one before it until you've done the final ending #.
5. See examples below:

$$\sum_{i=0}^n i = 0 + 1 + 2 + \dots + n$$

$$\sum_{i=0}^n i^2 = 0^2 + 1^2 + 2^2 + \dots + n^2$$

Practice: Evaluate the following summation symbols

$$\sum_{i=0}^4 i^2 = 0^2 + 1^2 + 2^2 + 3^2 + 4^2 = 0 + 1 + 4 + 9 + 16 = 30$$

Note that you are only plugging numbers into the variable i . For example:

$$\begin{aligned} \sum_{i=0}^4 3i^2 &= 3*0^2 + 3*1^2 + 3*2^2 + 3*3^2 + 3*4^2 = \\ &3*0 + 3*1 + 3*4 + 3*9 + 3*16 = 0 + 3 + 12 + 27 + 48 = 90 \end{aligned}$$

Also note that this is the same as if we had just multiplied $3*30 = 90$. In fact, you are allowed to pull a constant outside of a summation symbol.

$$\sum_{i=0}^4 3i^2 = 3 * \sum_{i=0}^4 i^2 = 3 * 30 = 90$$

You will need to know how to do these for Section 12.3 (Taylor Polynomials)

$$P_n(x) = \sum_{i=0}^n \frac{f^{(i)}(0)}{i!} x^i = \frac{f^{(0)}(0)}{0!} + \frac{f^{(1)}(0)}{1!} + \frac{f^{(2)}(0)}{2!} + \dots + \frac{f^{(n)}(0)}{n!}$$

Refresher on $n!$ (factorial). All you need to do is, starting with the number “ n ”, multiply by all the numbers going down from n to 1. So for example:

$$6! = 6*5*4*3*2*1 = 720$$

More Practice Problems:

$$\sum_{i=2}^6 i^2 = 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 4 + 9 + 16 + 25 + 36 = 90$$

$$\sum_{i=1}^3 (i+4)^2 = (1+4)^2 + (2+4)^2 + (3+4)^2 = 5^2 + 6^2 + 7^2 = 25 + 36 + 49 = 110$$

$$\sum_{i=1}^3 \frac{\sqrt{i-1}}{3} = \frac{\sqrt{1-1}}{3} + \frac{\sqrt{2-1}}{3} + \frac{\sqrt{3-1}}{3} = \frac{\sqrt{0}}{3} + \frac{\sqrt{1}}{3} + \frac{\sqrt{2}}{3} = 0 + \frac{1}{3} + \frac{\sqrt{2}}{3} \approx .8$$

Factoring

Have you ever gotten an answer you're sure is right, but that doesn't show up as one of the multiple choices on a test? One reason could be is that your answer is correct, but it's not in the algebraic form the test key was written in. Learning how to factor and simply algebraic expressions can be extremely useful in this scenario!

Factoring can be done when two or more terms in an expression contain the same variable, number, etc. For example:

$$2x + 5x = 7$$

$$x(2 + 5) = 7$$

$$x(7) = 7$$

$$x = 1$$

When you “factor out” a variable, it’s as if you were dividing each of those common terms by the variable. By the way, this last example is also called “adding like terms”.

Factoring can be useful to cancel out denominators. Notice how in this next example, each term in the numerator is divisible by 2.

$$\frac{2x + 4y + 8}{2} = \frac{\cancel{2}(x + 2y + 4)}{\cancel{2}} = x + 2y + 4$$

You can cancel out a denominator this way ONLY WHEN that number factors out of EVERY term on the top.

$$\frac{2x + 4y + 7}{2} = \frac{2(x + 2y) + 7}{2} \neq x + 2y + 7$$

Now that we’ve refreshed our memory of how this works, let’s try some harder problems. First notice that each term in the brackets has an e^{2x} . We factor that out, then see what’s left.

Simplify:

$$\left[12e^{2x} + 8xe^{2x} + (8x + 4)e^{2x} \right] = e^{2x} [12 + 8x + (8x + 4)]$$

Now we look to simplify what’s left inside the brackets.

$$e^{2x} [12 + 8x + (8x + 4)] = e^{2x} [16 + 16x]$$

Notice that each term left in the brackets is divisible by 16.

$$e^{2x} [12 + 8x + (8x + 4)] = e^{2x} [16 + 16x] = 16e^{2x} [1 + x]$$

Why is there a 1 left over when we factored out the first 16?

We can’t factor anything else. Why not?

Practice problems (simplify):

$$1. \frac{(x+4)^4(2x) - 4(x+4)^3 x^2}{(x+4)^8}$$

$$2. \frac{(x+4)^3(2x) - 4(x+4)^4 x^2}{(x+4)^8}$$

$$3. \frac{(x+4)^3(2x) - 4x^2}{(x+4)^8}$$

Take the derivative and factor as much as possible:

$$4. f(x) = \frac{e^{x^2} (4x+9)^7}{\ln(x^5)}$$

$$\text{Answer: } \frac{e^{x^2} (4x+9)^6 \left[2x \ln(x) (4x^2 + 9x + 14) - (4x+9) \right]}{5x(\ln(x))^2}$$