Intercepts and Asympototes

An intercept is where a function crosses a given axis.

Y-Intercept: This is where the equation crosses the y-axis.

In order to cross the y-axis, the x-coordinate must be zero. (Insert random graphs)
Think about it. When you look at the y-intercept in each graph, what is your x-value?
The y-intercept is the y-value when the x-coordinate is zero.

How to solve for the y-intercept?
As we said above, the y-intercept is the y-value when the x-coordinate is zero. It follows that if we want to find what the y-intercept of a function, we plug in zero for our x-value and calculate our y-value. The answer will be where the function crosses the y-axis. You will often write your answer as a coordinate pair: (0,y).

Sample Problem

1.) $f(x)=x^3+x^2+2$

The y-intercept is the function value when x equals zero so,

Y-intercept=$f(0)= (0)^3+(0)^2+2=2$ so the coordinate pair is (0,2).

X-Intercept: This is where the equation crosses the x-axis.

Similar to the y-intercept, in order to cross the x-axis, the y-coordinate must be zero. (Insert random graphs)

How to solve for the x-intercept?
As we said above, the x-intercept is the coordinate pair when the y-coordinate is zero. It follows that if we want to find what the x-intercept of a function, we plug in zero for our y-value and solve for our x-value. The answer will be where the function crosses the x-axis. The coordinate pair will have the following form: (x,0).

Sample Problem

2.) $f(x)= x^2+3x+2$

The x-intercept is the x-value when y equals zero so,

$f(x)=0=x^2+3x+2=(x+1)(x+2)$

For this equation to be true, x must equal -1 or -2, so our x-intercepts written as a coordinate pair would be (-1,0) and (-2,0).

Vertical Asymptotes

Vertical asymptotes look like a vertical line on the graph. They often occur at a value of x where the function is undefined.

Sample Problem: (Show a graph)

$f(x)=(x+2)/(x^2+2x-8)=(x+2)/((x+4)(x-2))$

This function cannot exist when x=-4 or when x=2 because that would make the denominator be zero, therefore this function has vertical asymptotes at x=-4 and x=2.

Horizontal Asymptotes

Horizontal asymptotes explain what a function is doing as the x-values get really big or really small. Functions often approach a given value as x approaches positive or negative infinity. Functions could also be getting continually bigger (approaching positive infinity) or continually smaller (approaching negative infinity) as the x-values increase or decrease. The horizontal asymptote is the function value (y-value) that is being approached, or even hit. To find this, plug increasingly large (or small) numbers in for x and see what the function is doing.

Sample problem: (Show a graph)

Find the horizontal asymptote of the following function:

$$y=\frac{x+2}{x^2+1}$$

The horizontal asymptote tells me, roughly, where the graph will go when *x* is really, really big. So I'll look at some very big values for *x*, some values of *x* very far from the origin:

x	$y=\frac{x+2}{x^2+1}$
–100 000	–0.0000099...
–10 000	–0.0000999...
–1 000	–0.0009979...
–100	–0.0097990...
–10	–0.0792079...
–1	0.5

0	2
1	1.5
10	0.1188118...
100	0.0101989...
1 000	0.0010019...
10 000	0.0001000...
100 000	0.0000100...

We can see as x gets smaller the function is getting closer to zero. As x gets bigger the function is also getting closer to zero. Therefore, the horizontal asymptote of this function is y=0.

Example Problems:

Calculate the y and x intercepts and any horizontal or vertical asymptotes.

1.) $f(x)=3x+5$
2.) $f(x)=(x-2)/(x^2-5x+4)$

Parent Functions

Based off the graph of a few functions, you can build almost any function. This can be done through translation, reflection, or stretching and compressing.

This site has graphs of all the parent functions:
http://faculty.gg.uwyo.edu/dueker/TALKS/Math%20Review/IG%20parent%20functons.pdf

Translation

Translation is sliding the graph (either up, down, left, or right).
Pull out a graphing calculator.
Let's look at the function $f(x)=x^2$.
What happens when we add 2 on the end: $f(x)= x^2+2$?
Every y-value (function value) that we would have gotten with $f(x)=x^2$ has increased by 2, so the whole graph has shifted up 2 places. If this was a negative 2 then the graph would have been shifted down 2. This is the case of any function. When you tack a number on at the end, it shifts the graph up or down that amount.

What if we added 2 to our x-value: $f(x)= (x+2)^2$?
Now, instead of changing our y-value, we're changing our x-value because the 2 is inside the parentheses, with the x-value, rather than at the end. In this case, it's as if each x value has been

increased by 2, so a x-value 2 below the one in the original function will yield the same point. Hence, the whole graph is slid to the left 2. What if the function had been changed to $f(x)= (x-2)^2$

Note: When changing y values "+" is up and "-" is down.

When changing x values "-" is right and "+" is left.

Reflection

A reflection is when a graph is flipped across the line, like a mirror image of itself.

To flip (reflect) a graph across the y-axis we want to change all the x-values (x) so they produce the function value of their negatives (-x). It follows that to reflect across the y-axis we can just change the x-values to (-x).

Note: Be careful to use your parentheses carefully so you are changing the x-value, not the y-value.

Sample: Write the equation of $f(x)= x^2+2$ reflected about the y-axis.

Insert (-x) for x so the new function becomes $g(x)= (-x)^2+2$.

To reflect a graph across the x-axis, you want to change all y-values to their negative. To do so, put a negative throughout the whole function.

Sample: Write the equation of $f(x)= x^2+2$ reflected about the x-axis.

Our new function is the negative of our old function so $g(x)=-f(x)= -x^2-2$.

Warning: Be sure you apply the negative throughout the entire function.

Stretching and Compressing

This is when a function becomes more compact or more spread out.

Pull out a graphing calculator again.

What happens to the function f(x)=x when you multiply it by 3: f(x)=3x?

You are moving through the y-values three times as fast, so the graph is compressed by 3 times the original.

What if you multiplied by a fraction?

These are the basic ways to manipulate parent functions. You can combine any of these together. Feel free to play with a graphing calculator and see how doing different things affects the parent graph. Here are some problems you can work through.

Graph the following functions:

1.) $f(x)=(x-3)^3 + 2$
2.) $f(x)=-2x^2 -2$
3.) $f(x)=-2(-x)^2 -2$