Algebra Review: Exponents and Logarithms

Week of 1/25/10

I. Exponents

Intro to Exponents:

1) Recall that $a^n = a \cdot a \cdot a \cdot a \ldots (n\ times)$

   →Example: $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

2) For a^0, we define it as $a^0 = 1$.

   →Examples: $5^0 = 1$, $(\frac{1}{5})^0 = 1$, $e^0 = 1$

3) For a^{-b}, we define it as $(1/a^b)$

   →Example: $5^{-3} = (1/5^3)$

Operations of Exponents:

1) Multiplication : $a^m \cdot a^n = a^{m+n}$

   -To multiply two exponential terms that have the same base, add their exponents.

   →Example: $3^2 \cdot 3^3 = 3^{3+2} = 3^5$

   -Do not add the exponents of terms with unlike bases.

   →Example: $2^2 \cdot 3^3 \neq 6^{3+2} \neq 6^5$

2) Division: $\frac{a^m}{a^n} = a^{m-n}$

   -To divide two exponential terms that have the same base, subtract their exponents.

   →Example: $\frac{7^6}{7^3} = 7^{6-3} = 7^3$

   -Do not subtract the exponents of terms with unlike bases

3) Exponents of Exponential Terms: $(a^m)^n = a^{mn}$

-To raise an exponential term to another exponent, multiply the two exponents.

→Example: $(2^3)^2 = 2^{2\cdot 3} = 2^6$

4) Products/quotients raised to exponents: $(ab)^m = (a^m b^m)$; $(\frac{a}{b})^m = \frac{a^m}{b^m}$

- To raise a product or a quotient to an exponent, apply the exponent to each individual part

→Examples: $(2x)^4 = 2^4 x^4 = 16x^4$; $(\frac{3}{x})^3 = \frac{3^3}{x^3} = \frac{27}{x^3}$

5) The FOIL method of multiplication

-To expand a binomial raised to a power, use the FOIL method (First, Outside, Inside, Last)

→Example: $(x+2)^2 = (x+2)(x+2) = x^2 + 2x + 2x + 4 = x^2 + 4x + 4$

Radicals:

Radicals are another form of exponents. Here's a helpful way to think about them:

$$\sqrt[n]{a} = a^{\frac{1}{n}};$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

It's often helpful in calculus to re-write radicals in exponential form. All exponent rules apply to radicals.

→Example: $(\sqrt{x+1})^2 = ((x+1)^{1/2})^2 = (x+1)^1 = x+1$

Special Cases:

- $\sin^2 x = (\sin x)^2$ $NOT\ sinx^2$
- $\sin^{-1} x = arcsinx\ NOT\ sinx^{-1}$

- this applies to the other trig functions as well
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II. The Logarithm

$$\text{If } b^c = a, \text{ then } \log_b a = c$$

A logarithm is just another way to write an exponent. If you want to find out what 5^2 is, you multiply two fives together to get 25. But if you want to find out which power you have to raise 5 to in order to get 25, you use a logarithm.

$\log_5 25 = ?$

The question you ask yourself when you look at this log is: To what power should I raise 5 in order to get 25? The answer is 2.

$\log_5 25 = 2$

Here's the general form of a logarithm:

$\log_b a = c$

The Common Log and the Natural Log

- Logarithms can have any base (*b*), but the 2 most common bases are 10 and e.
- Logs with bases of 10 are called common logs, and often the 10 is left out when a common log is written.
- →Example: $\log_{10} 100$ is the same as $\log 100$
- Logs with bases of *e* are known as natural logs. The shortened version of $\log_e x$ is $\ln x$.
- *e* is a constant with an approximate value of 2.71828. Don't let it scare you... it's just a number.

Simplifying Logarithms

The following rules for simplifying logarithms will be illustrated using the natural log, *ln*, but these rules apply to all logarithms.

1) **Adding logarithms** (with the same base)

$$\ln a + \ln b = \ln(a \cdot b)$$

Two logs of the same base that are added together can be consolidated into one log by *multiplying* the inside numbers.

→Example: $\ln 5 + \ln 4 = \ln(5 \cdot 4) = \ln 20$

2) **Subtracting logarithms** (with the same base)

$$\ln a - \ln b = \ln(a/b)$$

Similarly, two logs of the same base being substracted can be consolidated into one log by *dividing* the inside numbers.

→Example: $\ln 14 - \ln 2 = \ln(14/2) = \ln 7$

3) Exponents of logarithms
$\ln a^b = b \ln a$

If the inside number of the logarithm is raised to a power, you bring down the exponent as a coefficient.

→Example: $\ln 3^2 = 2 \ln 3$

4) Things that cancel
   - $\ln e = 1$
   - $\log_a a = 1$
   - $\ln 1 = 0$
   - $\log_a 1 = 0$
   - $e^{\ln x} = x$
   - $\ln e^x = x$